MATH 135

Final Exam Review

Simplify. Your answer should contain only positive exponents.

1) $\dfrac{2x^3y^4}{4y \cdot 3x^2} = \dfrac{2x^3y^4}{12x^2y} = \dfrac{xy^3}{6}$

2) $\sqrt[3]{-81u^7v^7} = \sqrt[3]{-27 \cdot 3 \cdot u^6 \cdot u \cdot v^6 \cdot v} = -3u^2v^2\sqrt[3]{3uv}$

3) $-7\sqrt{3x^2} \cdot 7\sqrt{5x^2} = -49\sqrt{15x^4} = -49x^2\sqrt{15}$

4) $\dfrac{2xy^2}{x^3} = \dfrac{2y^2}{x^2}$

Simplify each difference.

5) $(8a^3 - 6a + 5) - (4a^3 - 3a) = 4a^3 - 3a + 5$

Find each product.

6) $(7k^2 - 5k + 5)(k - 4)$ $= 7k^3 - 28k^2 - 5k^2 + 20k + 5k - 20$

$= 7k^3 - 33k^2 + 25k - 20$

Factor each.

7) $x^2 + 6x + 9 = 0$

$(x + 3)(x + 3) = 0$

$(x + 3)^2 = 0$

8) $x^3 - 27 = 0$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$a = x, b = 3$

$(x - 3)(x^2 + x(3) + 3^2) = 0$

$(x - 3)(x^2 + 3x + 9) = 0$

Simplify each expression.

9) $\frac{m-2}{m-1}+\frac{2}{2m^2}$ $LCD: 2m^2(m-1)$

$$\frac{2m^2(m-2)}{2m^2(m-1)}+\frac{2(m-1)}{2m^2(m-1)}=\frac{2m^3-4m^2+2m-2}{2m^2(m-1)}$$

$$=\frac{m^3-2m^2+m-1}{m^2(m-1)}$$

Simplify each expression.

10) $\dfrac{9x+36}{x-10} \cdot \dfrac{1}{x+4}$ $= \dfrac{9(x+4)}{(x-10)(x+4)} = \dfrac{9}{(x-10)}$

11) $\dfrac{n+6}{4} \div \dfrac{9}{4n-32}$ $= \dfrac{n+6}{4} \cdot \dfrac{4n-32}{9} = \dfrac{(n+6)\cdot 4(n-8)}{36}$

$= \dfrac{(n+6)\,(n-8)}{9} = \dfrac{n^2-2n-48}{9}$

Identify the center and radius of each. Then sketch the graph.

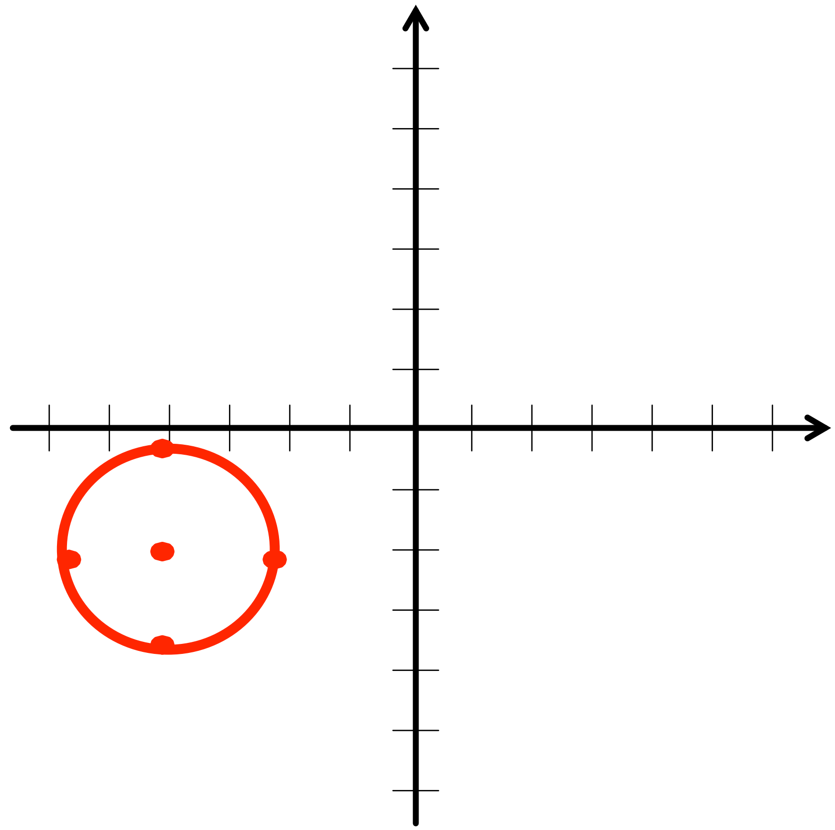
12) $x^2 + y^2 + 8x + 4y + 17 = 0$

$$(x^2 + 8x + 16) + (y^2 + 4y + 4) = -17 + 16 + 4$$

$$(x + 4)^2 + (y + 2)^2 = 3$$

$$center: (-4, -2)$$

$$radius = \sqrt{3} \approx 1.7$$



Find all roots.

13) $x^3 - 25x = 0$

$$x(x^2 - 25) = 0$$

$$x(x+5)(x-5) = 0$$

$x = 0$ $\quad x + 5 = 0$ $\quad x - 5 = 0$

$x = -5$ $\quad x = 5$

Solve each equation by factoring.

14) $x^2 - 3x = -2$

$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

$x - 2 = 0$ $\quad x - 1 = 0$

$x = 2$ $\quad x = 1$

Solve each equation. Remember to check for extraneous solutions.

15) $\frac{1}{2b} + \frac{5b-30}{4b} = \frac{3}{4b}$ $\quad LCD: 4b$

$$\frac{2}{4b} + \frac{5b-30}{4b} = \frac{3}{4b}$$

$$\frac{5b-28}{4b} = \frac{3}{4b}$$

$$5b - 28 = 3$$

$$5b = 31$$

$$b = \frac{31}{5}$$

16) $5 = \sqrt{3a-5}$

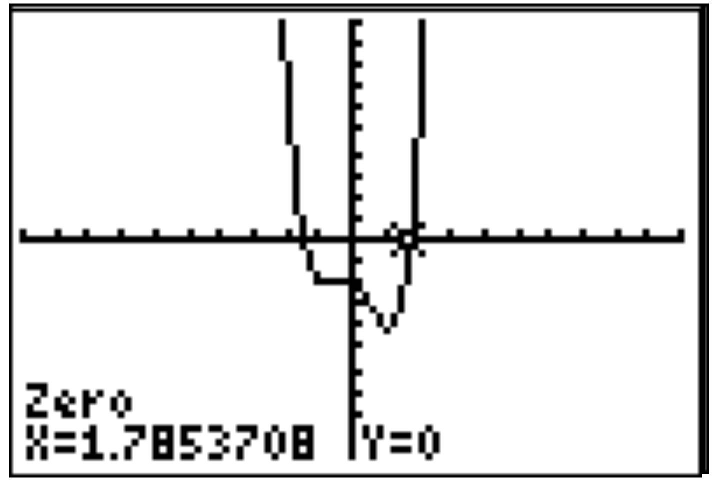
$$25 = 3a - 5$$

$$30 = 3a$$

$$10 = a$$

Approximate the real zeros of each function to the nearest tenth.

17) $f(x) = x^4 - 2x^2 - x - 2$



$zeros: -1.5, 1.8$

Solve each compound inequality and graph its solution.

18) $-36 < -9k < 27$

$4 > k > -3$

$-3 < k < 4$

Write the slope-intercept form of the equation of the line through the given point with the given slope.

19) through: $(-3, 0)$, slope $= 2$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = 2(x - (-3))$$

$$y = 2(x + 3) \qquad y = 2x + 6$$

Write the slope-intercept form of the equation of the line through the given points.

20) through: $(-3, -3)$ and $(5, 3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{3 - (-3)}{5 - (-3)} = \frac{6}{8} = \frac{3}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{3}{4}(x - 5)$$

$$y - 3 = \frac{3}{4}x - \frac{15}{4}$$

$$y = \frac{3}{4}x - \frac{3}{4}$$

Perform the indicated operation.

21) $g(x) = 3x + 5$
$h(x) = 3x$
Find $\left(\frac{g}{h}\right)(x)$

$$\frac{g}{h}(x) = \frac{3x+5}{3x}, x \neq 0$$

22) $g(x) = 3x^2 + 3x$
$h(x) = x + 5$
Find $(g \circ h)(x) = g(h(x))$

$$g(h(x)) = 3(x+5)^2 + 3(x+5)$$
$$= 3(x^2 + 10x + 25) + 3x + 15$$
$$= 3x^2 + 30x + 75 + 3x + 15$$
$$= 3x^2 + 33x + 90$$

Find the inverse of each function.

23) $g(x) = 2 - x^3$ *switch the x & y*

$x = 2 - y^3$ *solve for y*

$x - 2 = -y^3$

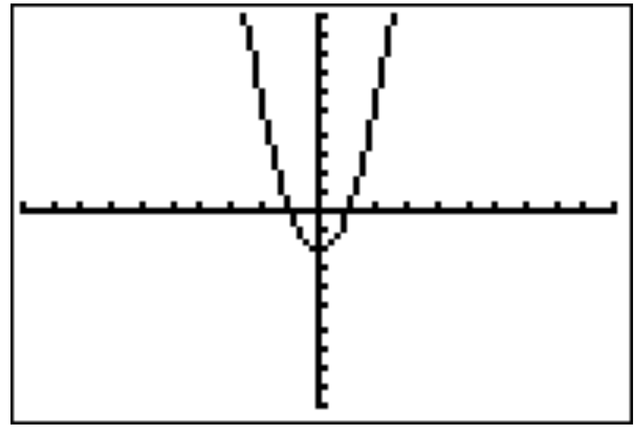
$-x + 2 = y^3$

$\sqrt[3]{-x+2} = y$

$g^{-1}(x) = \sqrt[3]{-x+2}$

Describe the end behavior of each function.

24) $f(x) = 2x^2 - 2$

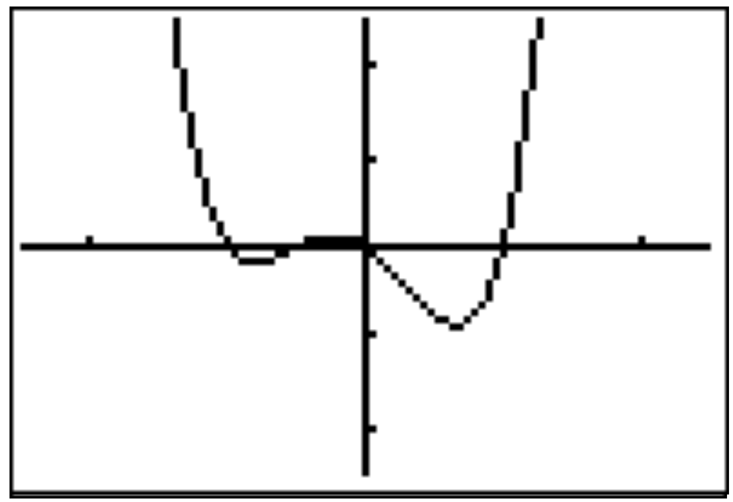


up to the left $\quad f(x) \to \infty \text{ as } x \to -\infty$

up to the right $\quad f(x) \to \infty \text{ as } x \to \infty$

Find all zeros.

25) $f(x) = 2x^4 + x^3 - 2x^2 - x$



factor by grouping

$$(2x^4 - 2x^2) + (x^3 - x) = 0$$

$$2x^2(x^2 - 1) + x(x^2 - 1) = 0$$

$$(x^2 - 1)(2x^2 + x) = 0$$

$$(x + 1)(x - 1) \cdot x(2x + 1) = 0$$

$$x(2x + 1)(x + 1)(x - 1) = 0$$

$x = 0 \qquad 2x + 1 = 0 \qquad x + 1 = 0 \qquad x - 1 = 0$

$\qquad\qquad x = -1/2 \qquad x = -1 \qquad x = 1$

State the maximum number of turns the graph of each function could make. Approximate each real zero to the nearest tenth. Approximate the relative minima and relative maxima to the nearest tenth.

26) $f(x) = x^4 - 2x^2 + 2x - 4$

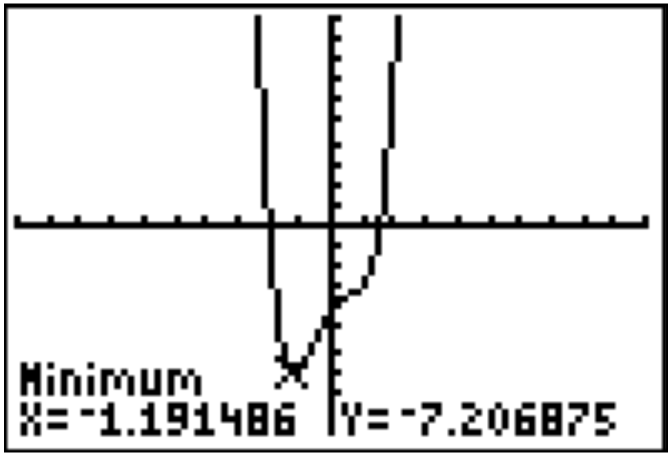
$max\ turns = degree - 1$

$max\ turns = 3$

$zeros: -2, 1.5$

$relative\ min: (-1.2, -7.2)$

$relative\ max: \ none$



State the possible rational zeros for each function.
Then find all rational zeros. One zero has been given.

27) $f(x) = x^3 - 11x^2 + 27x + 15;\ 5$

$$possible\ roots = \frac{factors\ of\ constant}{factors\ of\ leading\ coefficient}$$

$$PR = \frac{\pm 1, \pm 3, \pm 5, \pm 15}{\pm 1} = \pm 1, \pm 3, \pm 5, \pm 15$$

5	1	−11	27	15
	↓	5	−30	−15
	1	−6	−3	0

$x^2 - 6x - 3$

5 *is the only rational zero*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36 + 12}}{2} = \frac{6 \pm \sqrt{48}}{2}$$

$$= \frac{6 \pm 4\sqrt{3}}{2} = 3 \pm 2\sqrt{3}$$

Identify the holes, vertical asymptotes, and horizontal asymptote of each. Then sketch the graph.

28) $f(x) = \dfrac{x^3 + 5x^2 + 4x}{3x^3 + 21x^2 + 36x} = \dfrac{x(x^2 + 5x + 4)}{3x(x^2 + 7x + 12)} = \dfrac{x(x+4)(x+1)}{3x(x+4)(x+3)}$

$f(x) = \dfrac{(x+1)}{3(x+3)} = \dfrac{x+1}{3x+9}$

$holes\ @\ x = 0\ \&\ x + 4 = 0$

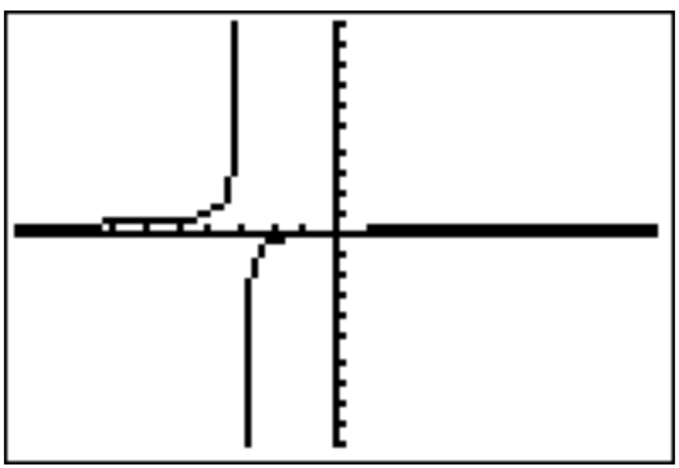
$x = -4$

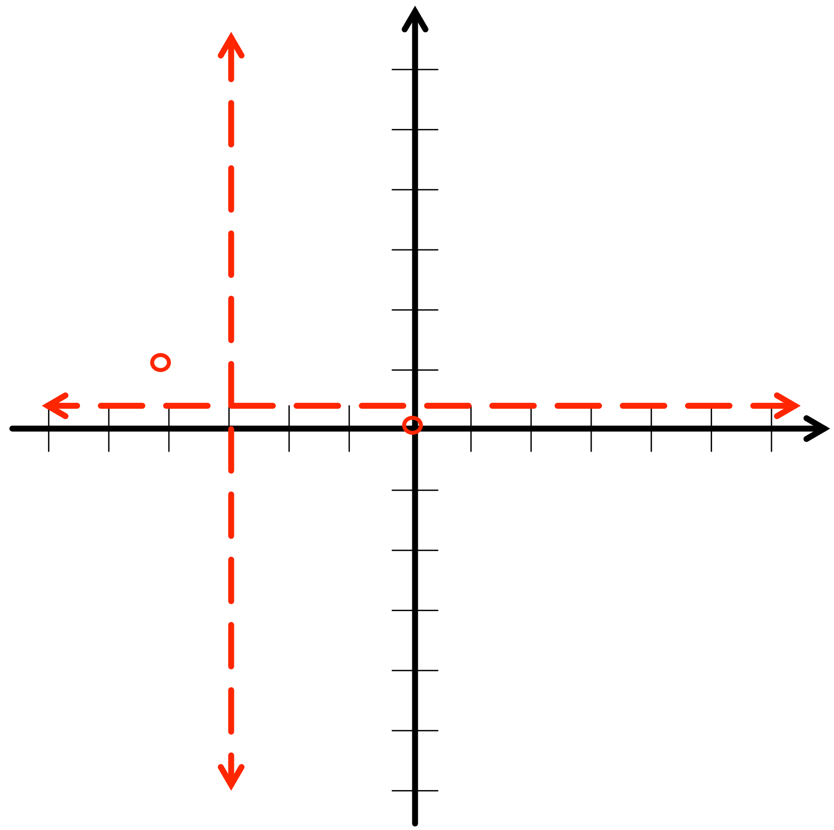
$VA:\ 3x + 9 = 0$

$3x = -9$

$x = -3$

$HA:\ y = \dfrac{1}{3}$





Solve each equation.

29) $5^{-2n} = 1$

$$\log 5^{-2n} = \log 1$$

$$-2n \log 5 = \log 1 \qquad n = \frac{\log 1}{-2\log 5} = 0$$

Solve each equation. Round your answers to the nearest ten-thousandth.

30) $18^{6m} + 9 = 53$

$$18^{6m} = 44$$

$$\log 18^{6m} = \log 44$$

$$6m \log 18 = \log 44 \qquad m = \frac{\log 44}{6\log 18} \approx 0.2182$$

Condense each expression to a single logarithm.

31) $8\log_5 8 + 2\log_5 11$

$$\log_5 8^8 + \log_5 11^2 = \log_5(8^8 \cdot 11^2) = \log_5 2030043136$$

Evaluate each expression.

32) $\log_7 49 = x$

$$7^x = 49$$

$$7^x = 7^2$$

$$x = 2$$

Rewrite each equation in exponential form.

33) $\log_6 \frac{1}{36} = -2$

$$6^{-2} = \frac{1}{36}$$

Solve each equation.

34) $\log_2 3x^2 + \log_2 6 = \log_2 72$

$$\log_2(18x^2) = \log_2 72$$

$$18x^2 = 72$$

$$x^2 = 4$$

$$x = \pm 2$$

Find a positive and a negative coterminal angle for each given angle.

35) $-80°$ *To find coterminal angles, add and subtract* $360°$

$$\theta_+ = -80° + 360° = 280°$$

$$\theta_- = -80° - 360° = -440°$$

Convert each degree measure into radians.

36) $280°$

$$280° \cdot \frac{\pi}{180°} = \frac{280°\pi}{180°} = \frac{14\pi}{9}$$

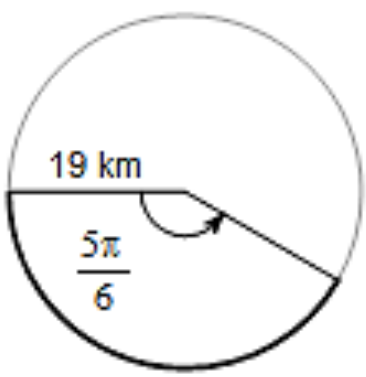
Convert each radian measure into degrees.

37) $-\frac{29\pi}{18}$

$$-\frac{29\pi}{18} \cdot \frac{180°}{\pi} = -\frac{5220°\pi}{18\pi} = -290°$$

Find the length of each arc.

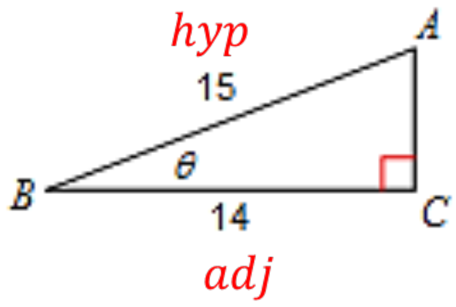
38)



$$s = r\theta = (19km)\left(\frac{5\pi}{6}\right) = \frac{95\pi}{6}km \approx 49.74km$$

Find the measure of each angle indicated. Round to the nearest tenth.

39)

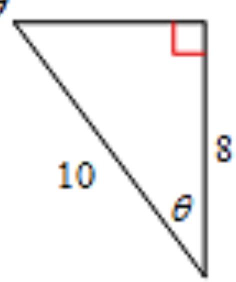


$$\cos\theta = \frac{14}{15}$$

$$\theta = \cos^{-1}\left(\frac{14}{15}\right) \approx 21.0° \; or \; 0.4 \; radians$$

Find the value of the trig function indicated.

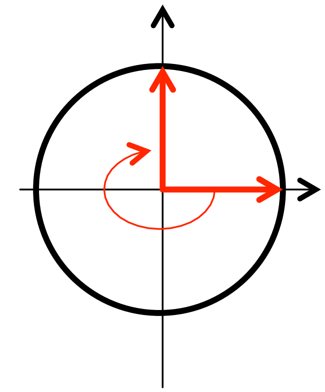
40) $\cos\theta$



$$\cos\theta = \frac{adj}{hyp} = \frac{8}{10} = \frac{4}{5}$$

Find the exact value of each trigonometric function.

41) $\sin -\frac{3\pi}{2} = 1$



Solve each equation for $0 \le \theta < 2\pi$.

42) $4 = 4 - \frac{1}{3} \cdot \cos\ \theta$

$$0 = -\frac{1}{3}\cos\theta$$

$$0 = \cos\theta$$

$$\theta = \frac{\pi}{2}\ and\ \frac{3\pi}{2}$$

Using radians, find the amplitude and period of each function.

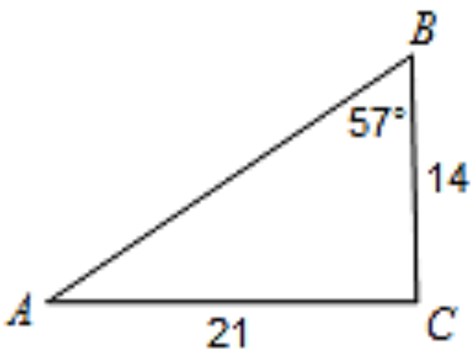
43) $y = \frac{1}{6} \cdot \cos\left(3\theta + \frac{\pi}{3}\right)$

$$amplitude = |a| = \left|\frac{1}{6}\right| = \frac{1}{6}$$

$$period = \frac{2\pi}{b} = \frac{2\pi}{3}$$

Solve each triangle. Round your answers to the nearest tenth.

44)



This is a SSA triangle, and θ is acute, so begin by checking to see if opp $\geq$ adj. It is, so there is one triangle.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{21}{\sin 57°} = \frac{14}{\sin A}$$

$$21 \sin A = 14 \sin 57°$$

$$\sin A = \frac{14 \sin 57°}{21}$$

$$A = \sin^{-1}\left(\frac{14 \sin 57°}{21}\right)$$

$$A \approx 34.0°$$

$$C = 180° - 57° - 34.0°$$

$$C = 89°$$

A =	34.0°	a =	14
B =	57°	b =	21
C =	89°	c =	25.0

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{21}{\sin 57°} = \frac{c}{\sin 89°}$$

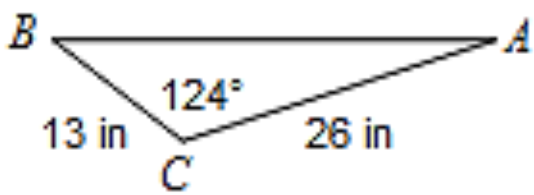
$$21 \sin 89° = c \sin 57°$$

$$c = \frac{21 \sin 89°}{\sin 57°}$$

$$c \approx 25.0$$

Solve each triangle. Round your answers to the nearest tenth.

45)



A = 17.9° a = 13

B = 38.1° b = 26

C = 124° c = 35.0

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 13^2 + 26^2 - 2(13)(26)\cos 124°$$

$$c^2 \approx 1223.0$$

$$c \approx 35.0$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{35}{\sin 124°} = \frac{13}{\sin A}$$

$$35\sin A = 13\sin 124°$$

$$\sin A = \frac{13\sin 124°}{35}$$

$$A = \sin^{-1}\left(\frac{13\sin 124°}{35}\right)$$

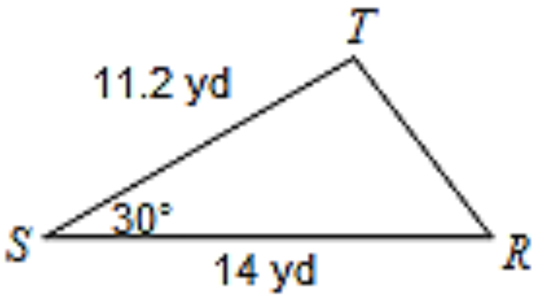
$$A \approx 17.9°$$

$$B = 180° - 124° - 17.9°$$

$$B = 38.1°$$

Find the area of each triangle to the nearest tenth.

46)

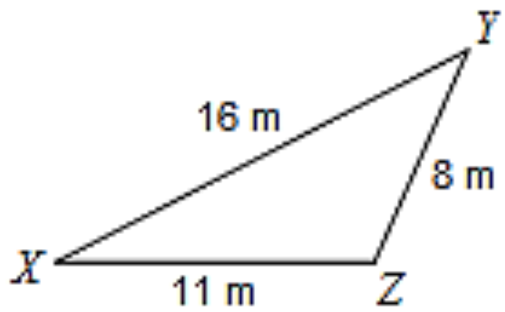


$$A = \frac{1}{2} s_1 s_2 \sin\theta$$

$$A = \frac{1}{2}(11.2)(14)\sin 30°$$

$$A \approx 39.2\ square\ yards$$

47)



$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2} = \frac{8+11+16}{2} = 17.5$$

$$A = \sqrt{17.5(17.5-8)(17.5-11)(17.5-16)}$$

$$= \sqrt{17.5(9.5)(6.5)(1.5)}$$

$$= \sqrt{1620.9375}$$

$$\approx 40.3\ square\ meters$$

Identify the vertex, focus, and directrix of each. Then sketch the graph.

48) $y = 3x^2 - 30x + 71$

$$y - 71 = 3x^2 - 30x$$

$$75 + y - 71 = 3(x^2 - 10x + 25)$$

$$y + 4 = 3(x - 5)^2$$

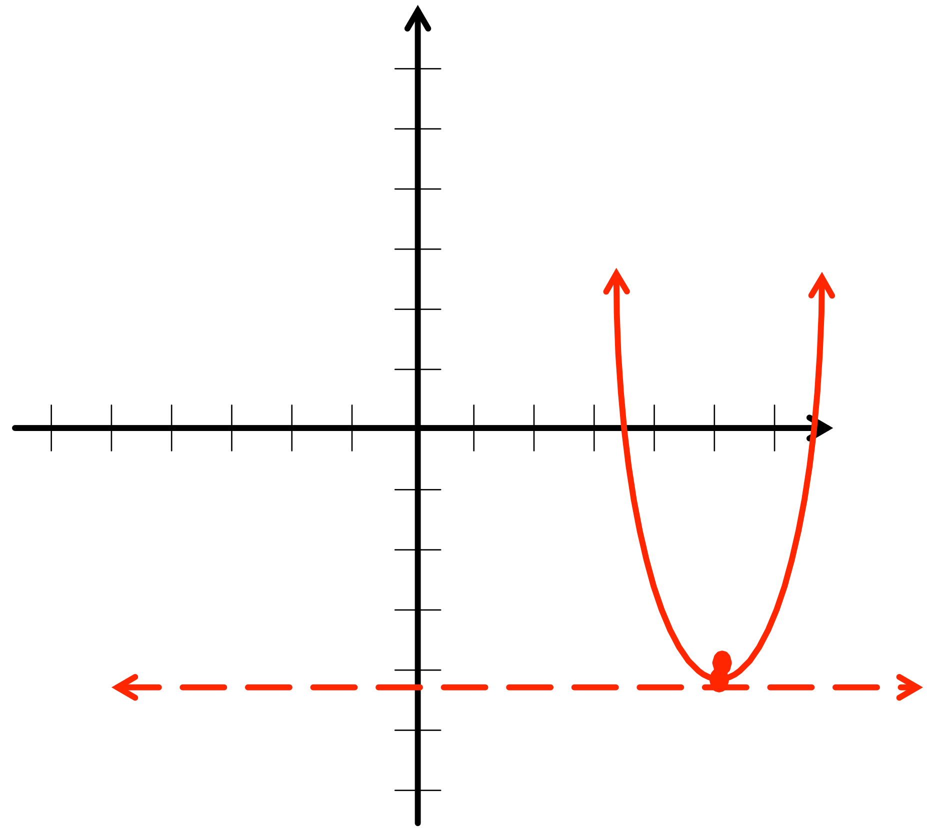
$$vertex: (5, -4)$$

The parabola will open up.

$$p = \frac{1}{4a} = \frac{1}{4(3)} = \frac{1}{12}$$

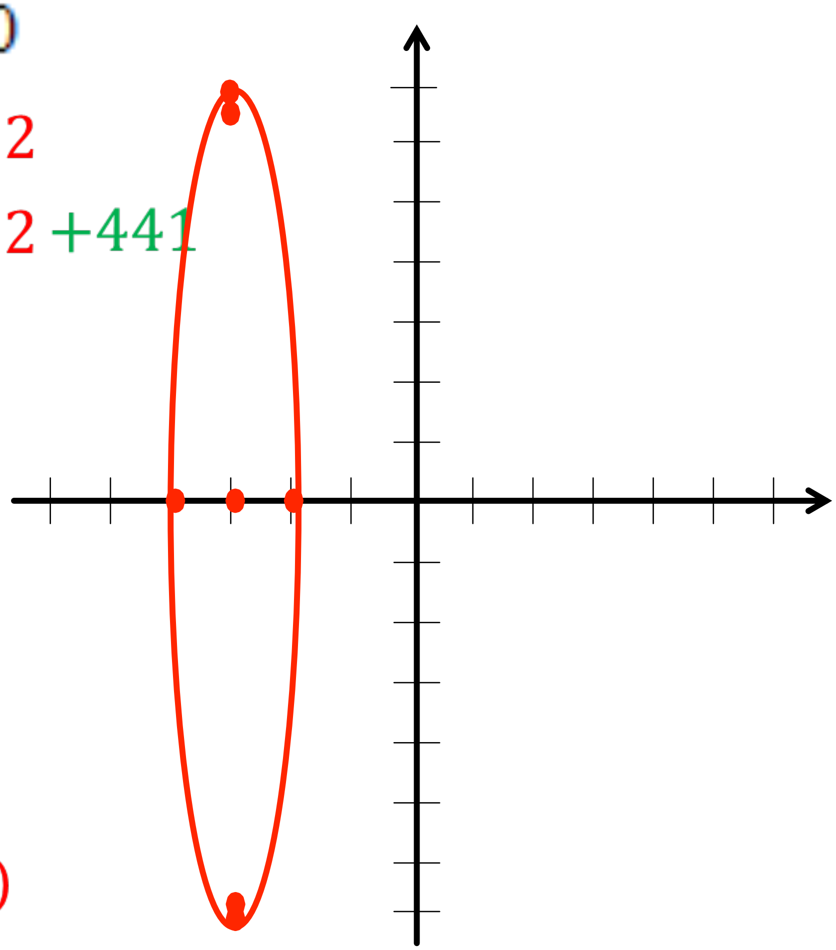
$$focus: \left(5, -3\tfrac{11}{12}\right)$$

$$directrix: y = -4\tfrac{1}{12}$$



Identify the center, vertices, co-vertices, and foci of each. Then sketch the graph.

49) $49x^2 + y^2 + 294x + 392 = 0$



$$(49x^2 + 294x \quad) + y^2 = -392$$

$$49(x^2 + 6x \ +9 \ ) + y^2 = -392 + 441$$

$$49(x+3)^2 + y^2 = 49$$

$$\frac{(x+3)^2}{1} + \frac{y^2}{49} = 1$$

$$center: (-3, 0)$$

$$left \,\&\, right: 1$$

$$up \,\&\, down: 7$$

$$vertices: (-3, 7) \,\&\, (-3, -7)$$

$$c = \sqrt{|a^2 - b^2|} = \sqrt{|1^2 - 7^2|}$$

$$= \sqrt{|1 - 49|} = \sqrt{|-48|} = \sqrt{48} = 4\sqrt{3} \approx 6.9$$

$$foci: \left(-3, 4\sqrt{3}\right) \,\&\, \left(-3, -4\sqrt{3}\right)$$

Identify the vertices and foci of each. Then sketch the graph.

50) $-9x^2 + 4y^2 - 18x - 45 = 0$

$4y^2 - (9x^2 + 18x \quad) = 45$

$4y^2 - 9(x^2 + 2x + 1) = 45 - 9$

$4y^2 - 9(x+1)^2 = 36$

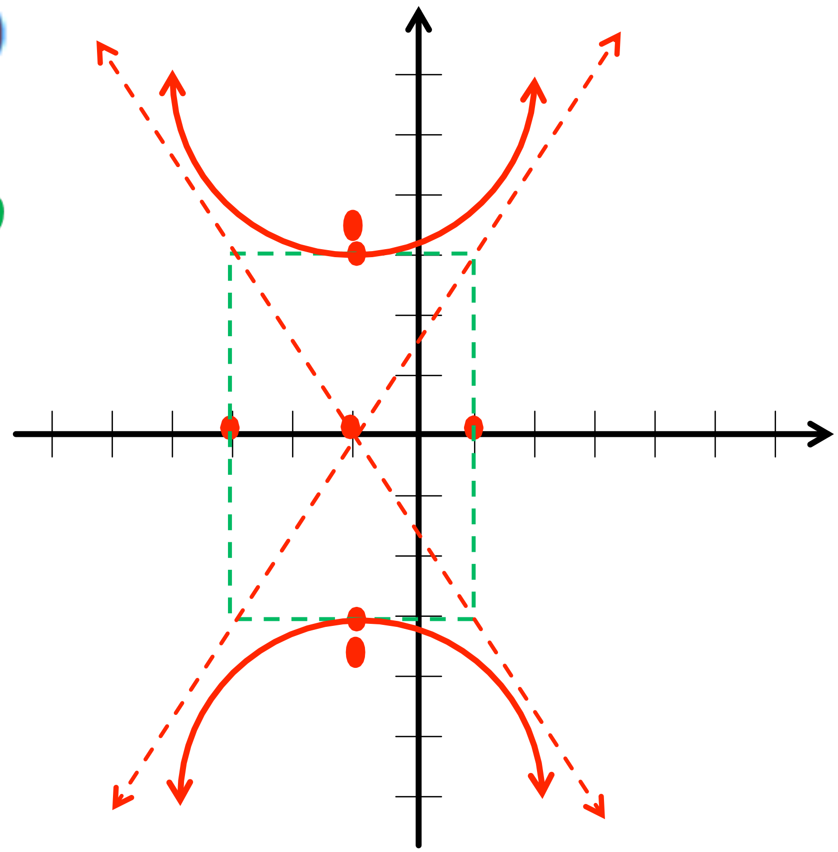
$$\frac{y^2}{9} - \frac{(x+1)^2}{4} = 1$$

center: $(-1, 0)$

left & right: 2

up & down: 3

The parabolas will open up and down.



vertices: $(-1, 3)$ & $(-1, -3)$

$c = \sqrt{a^2 + b^2} = \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13} \approx 3.6$

foci: $(-1, \sqrt{13})$ & $(-1, -\sqrt{13})$

SOLUTIONS

1) $\frac{xy^3}{6}$

2) $-3u^2v^2\sqrt[3]{3uv}$

3) $-49x^2\sqrt{15}$

4) $\frac{2y^2}{x^2}$

5) $4a^3 - 3a + 5$

6) $7k^3 - 33k^2 + 25k - 20$

7) $(x+3)^2 = 0$

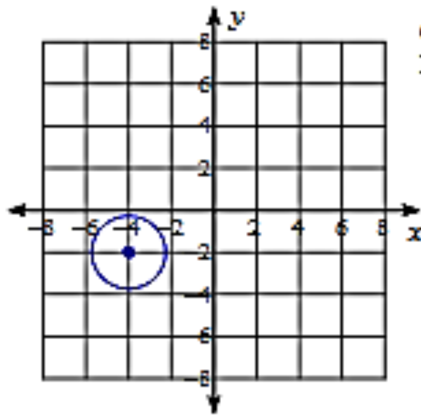
8) $(x-3)(x^2+3x+9) = 0$

9) $\frac{m^3 - 2m^2 + m - 1}{m^2(m-1)}$

10) $\frac{9}{x-10}$

11) $\frac{(n+6)(n-8)}{9}$

12)



Center: $(-4, -2)$
Radius: $\sqrt{3}$

13) $\{0, -5, 5\}$

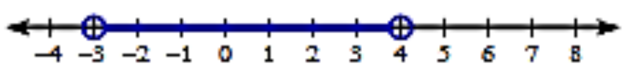
14) $\{2, 1\}$

15) $\left\{\frac{31}{5}\right\}$

16) $\{10\}$

17) Real Zeros: $-1.5, 1.8$

18) $-3 < k < 4$:



19) $y = 2x + 6$

20) $y = \frac{3}{4}x - \frac{3}{4}$

21) $\frac{3x+5}{3x}$

22) $3x^2 + 33x + 90$

23) $g^{-1}(x) = \sqrt[3]{-x+2}$

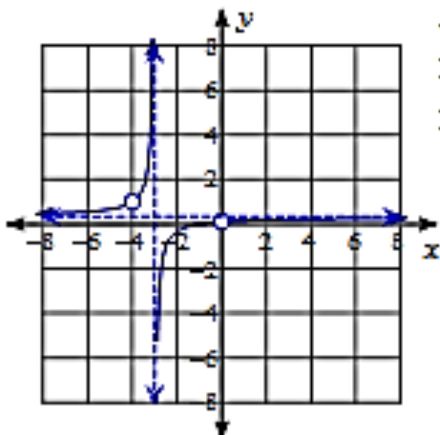
24) $f(x) \to +\infty$ as $x \to -\infty$
$f(x) \to +\infty$ as $x \to +\infty$

25) $\left\{0, 1, -\frac{1}{2}, -1\right\}$

26) Max # Turns: 3
Real Zeros: -2, 1.5
Minima: $(-1.2, -7.2)$
Maxima: None

27) Possible rational zeros: $\pm 1, \pm 3, \pm 5, \pm 15$
Rational zeros: $\{5\}$

28)



Vertical Asym.: $x = -3$
Holes: $x = 0, x = -4$
Horz. Asym.: $y = \frac{1}{3}$

29) $\{0\}$

30) 0.2182

31) $\log_5 \left(11^2 \cdot 8^8\right)$

32) 2

33) $6^{-2} = \frac{1}{36}$

34) $\{2, -2\}$

35) 280° and $-440°$

36) $\frac{14\pi}{9}$

37) -290°

38) $\frac{95\pi}{6}$ km

39) 21°

40) $\frac{4}{5}$

41) 1

42) $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

43) Amplitude: $\frac{1}{6}$

Period: $\frac{2\pi}{3}$

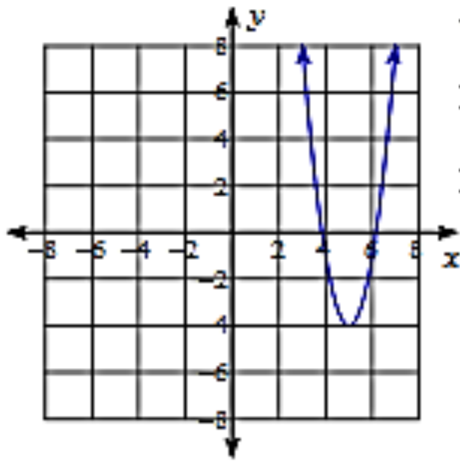
44) $m\angle C = 89°$, $m\angle A = 34°$, $c = 25$

45) $m\angle A = 18°$, $m\angle B = 38°$, $c = 35$ in

46) 39.2 yd^2

47) 40.3 m^2

48)

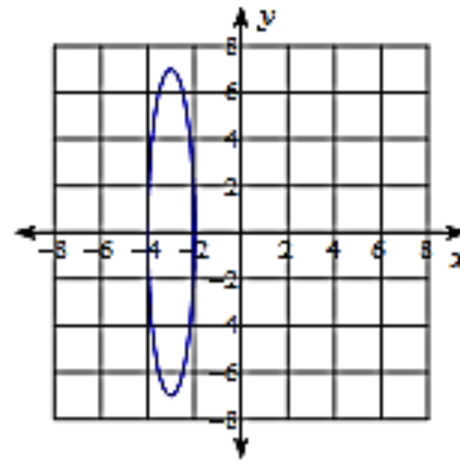


Vertex: $(5, -4)$

Focus: $\left(5, -\frac{47}{12}\right)$

Directrix: $y = -\frac{49}{12}$

49)



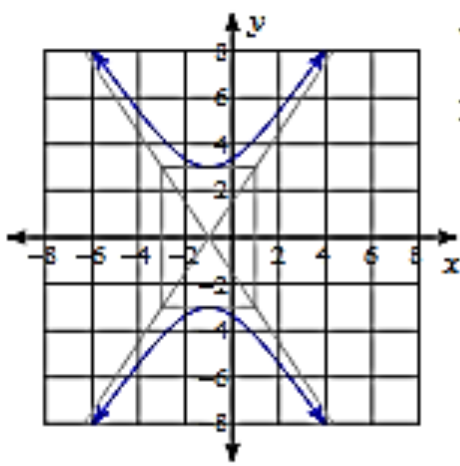
Center: $(-3, 0)$

Vertices: $(-3, 7)$
$(-3, -7)$

Co-vertices: $(-2, 0)$
$(-4, 0)$

Foci: $(-3, 4\sqrt{3})$
$(-3, -4\sqrt{3})$

50)



Vertices: $(-1, 3)$
$(-1, -3)$

Foci: $(-1, \sqrt{13})$
$(-1, -\sqrt{13})$